

Christian Schnell

Course: Kodaira dimension of algebraic fiber spaces over abelian varieties (after Cao and Paun)

Abstract:

Consider a morphism from a smooth projective variety to an abelian variety (over the field of complex numbers). After reviewing what is known about the pushforward of the canonical bundle under such a morphism, we will try to extend these results to the case of pluricanonical bundles (= the tensor powers of the canonical bundle). Along the way, we will learn about three important tools: generic vanishing theory; Viehweg's cyclic covering trick; and some new results from complex analysis about metrics with singularities. As an application, we will discuss the proof of Iitaka's conjecture (about the subadditivity of the Kodaira dimension in algebraic fiber spaces) over abelian varieties, following Cao and Paun.

References:

Junyan Cao and Mihai Paun, "Algebraic fiber spaces over abelian varieties", arXiv:1504.01095

Christopher Hacon, Mihnea Popa, and Christian Schnell, "Algebraic fiber spaces over abelian varieties: around a recent theorem by Cao and Paun", arXiv:1611.08768

Christian Schnell, "Lectures on the generic vanishing theorem and its applications", <http://www.math.stonybrook.edu/~cschnell/pdf/notes/generic-vanishing.pdf>

Mihai Paun, "Singular hermitian metrics and positivity of direct images of pluricanonical bundles", arXiv:1606.00174

Rita Pardini

Course: Paracanonical systems

Abstract: Given an irregular complex projective manifold  $X$ , its paracanonical system is the family of all effective divisors algebraically equivalent to a canonical divisor. Paracanonical systems have been an object of study for a long time and have been motivation for the theory of generic vanishing ([3], [4], [6], [7], [8]). I will recall general facts on continuous systems and

deformations of effective divisors (including recent results from [5], [12]) and show how these can be used to study the geometry of the paracanonical system when  $X$  has maximal Albanese dimension ([3], [4], [8], [11], [12]).

I will also introduce the so-called "eventual paracanonical map" ([3], [10]), a new intrinsic invariant of varieties of Albanese general type whose canonical bundle has positive Euler characteristic, and discuss its geometry and some examples.

#### References:

- [1] M.A. Barja, R. Pardini, L. Stoppino, "Linear systems on irregular varieties". arXiv: AG 1606.03290
- [2] M.A. Barja, R. Pardini, L. Stoppino, "The eventual paracanonical map of a variety of maximal Albanese dimension". arXiv: AG 1606.03290
- [3] A. Beauville, "Annulation du  $H^1$  et systèmes paracanoniques sur les surfaces, *J. Reine Angew. Math.* 388 (1988), 149–157.
- [4] A. Beauville, "Annulation du  $H^1$  pour les fibrés en droites plats, *Complex algebraic varieties (Bayreuth, 1990)*, 1–15, *Lecture Notes in Math.*, 1507, Springer, Berlin, 1992.
- [5] A. Castorena, G.P. Pirola, "Some results on deformations of sections of vector bundles". *Collectanea Mathematica* 68 no. 1, 2017, 9–20.
- [6] F. Catanese, "Moduli of surfaces of general type". *Algebraic geometry—open problems (Ravello, 1982)*, 90–112, *Lecture Notes in Math.*, 997, Springer, Berlin–New York, 1983.
- [7] F. Catanese, "Moduli and classification of irregular Kaehler manifolds (and algebraic varieties) with Albanese general type fibrations", *Invent. Math.* 104 (2), 1991, 263–289.
- [8] M.Green, R.Lazarsfeld, "Deformation theory, generic vanishing theorems, and some conjectures of Enriques, Catanese and Beauville", *Invent. Math.* 90, 1987, 416–440.
- [9] M.Green, R.Lazarsfeld, "Higher obstructions to deforming cohomology groups of line bundles", *J. Amer. Math. Soc.* 4 (1991), no. 1, 87–103.
- [10] Z. Jiang, "Some results on the eventual paracanonical maps". arXiv: AG 1611.07141
- [11] R. Lazarsfeld, M. Popa, "Derivative complex, BGG correspondence and numerical inequalities for compact Kaehler manifolds", *Invent. Math.* 182 (2010), 605–633.
- [12] M. Mendes Lopes, R. Pardini, G.P. Pirola, "Continuous families of divisors, paracanonical systems, and a new inequality for varieties of maximal Albanese dimension". *Geo. Topol.* 17, 1205–1223 (2013)
- [13] D. Mumford, *Lectures on Curves on an Algebraic Surface.* (AM–59). Princeton University Press, New Jersey (1966)
- [14] R. Pardini, "Abelian covers of algebraic varieties", *J. reine angew. Math.* 417 (1991), 191–213

Miguel Angel Barja

Course title: Numerical invariants of continuous systems.

Abstract: Given a line bundle on an irregular variety, we will study qualitative and numerical properties of continuous systems associated to line bundles. First of all, we will study properties which are eventual under the process of multiplication maps, such as the continuous resolution of base points or the eventual factorization of continuous systems. Then we will introduce new numerical invariants associated to continuous systems: the continuous rank and the eventual degree. We will show how the continuous rank can be extended to a real convex function and we will study its regularity properties. We will state and prove several Clifford–Severi inequalities: lower bounds of the volume of continuous systems depending on the continuous rank and other numerical invariants. Finally, we will study some limit cases of the Clifford–Severi inequalities.

Bibliography:

- R. Pardini, "The Severi inequality  $K^2 \geq 4\chi$  for surfaces of maximal Albanese dimension". *Invent. Math.* 159 3 (2005), 669–672.
- M.A. Barja, "Generalized Clifford Severi inequality and the volume of irregular varieties". *Duke Math. J.* 164 (2015), no. 3, 541–568.
- M.A. Barja, R. Pardini, L. Stoppino, "Surfaces on the Severi line". *Journal de Mathématiques Pures et Appliquées*, (2016), no. 5, 734–743.
- M.A. Barja, R. Pardini, L. Stoppino, "Linear systems on irregular varieties". arXiv: AG 1606.03290